OUTER WORLD DRONES

PROJECT TITANWING

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INTRODUCTION

Unmanned aerial vehicles (UAVs), commonly known as drones, have seen a meteoric rise in both professional and recreational applications on Earth. Their ability to navigate challenging terrains, capture breathtaking aerial views, and assist in critical missions, such as search and rescue or scientific data collection, underscores their invaluable utility. As our exploratory ambitions extend beyond our home planet, the prospect of deploying these versatile machines on other celestial bodies becomes an increasingly tangible reality.

A testament to this extraterrestrial ambition is NASA's Ingenuity, a small helicopter that accompanied the Perseverance rover to Mars. Ingenuity's successful flights on Martian terrain have proven that aerial exploration is feasible on planets other than Earth, opening a realm of possibilities for aerial reconnaissance on distant worlds.

While Mars has piqued significant interest, Saturn's largest moon, Titan, emerges as an even more intriguing candidate for aerodynamic explorations. Unlike Mars, with its thin atmosphere, or the Moon, with no atmosphere at all, Titan boasts a dense atmosphere, richer than that of Earth. Coupled with its landscape of lakes, rivers, dunes, and mountains, Titan is a trove of scientific mysteries waiting to be unraveled.

The selection of Titan over other celestial bodies for this study is informed by these unique attributes. While other planets and moons present their own sets of challenges and curiosities, Titan's atmospheric density offers the tantalizing possibility of sustained aerodynamic flight, a scenario markedly different from the brief hops achievable on Mars.

This report delves into the heart of this challenge. Through rigorous simulation, we contrast the flight dynamics of a quadcopter on Earth with its potential behavior on Titan. Our aim is to provide a comprehensive understanding of the opportunities and challenges of quadcopter flight on Titan, setting the groundwork for potential future missions.

By building on the pioneering steps of Ingenuity on Mars and extending our gaze to the dense atmosphere of Titan, this study seeks to push the boundaries of what we believe is possible for UAVs in our solar system.

THE TITANWING PROJECT

Motive:
The primary impetus for this research stems from the broader aim of expanding our horizons of extraterrestrial exploration. As the success of rovers has ushered in a new era of surface exploration, the next frontier lies in understanding and mastering the aerial dimension. While surface rovers like Perseverance provide invaluable data, their range and perspective are inherently limited. Aerial vehicles, with their ability to cover vast areas and access hard-to-reach terrains, promise a new vantage point to study alien landscapes. Titan, with its unique atmospheric properties and varied terrain, offers an unprecedented opportunity to employ these aerial assets, potentially revolutionizing our understanding of its complex environmental and geological systems.

Drone Configurations:
When considering UAV designs suitable for Titan, several configurations emerge as contenders:

1. **Standard Quadcopter**: This familiar design consists of four rotors and offers a balance of stability and maneuverability. Its symmetrical layout ensures redundancy, which could be crucial in an extraterrestrial environment where reliability is paramount.
2. **Hexacopter**: With six rotors, this configuration offers increased lift capacity, which might be advantageous given the denser atmosphere of Titan. The added rotors also provide greater redundancy, enhancing reliability.
3. **Tilt-Rotor**: A design that allows the rotors to tilt can transition between vertical and horizontal flight. This could be especially useful for fast traversal over Titan's vast landscapes, offering a blend of the hover capability of a drone with the speed and efficiency of an airplane.
4. **Single Rotor Helicopter**: Similar to the Mars Ingenuity, this design is simplistic and might be easier to control. However, its single point of failure and potential difficulties in generating sufficient lift in Titan's dense atmosphere make it a less favorable option.
5. **Gas Enveloped Drones**: Given the lower gravity and dense atmosphere of Titan, drones that leverage buoyancy, akin to blimps or balloons, with propellers for maneuverability might be an interesting proposition.

Selection for Titan:
After analyzing the various configurations, the **Hexacopter** emerges as the prime candidate for deployment on Titan. Its multiple rotors grant it the lift required to navigate Titan's dense atmosphere efficiently. The redundancy offered by six rotors ensures that even in the event of a rotor malfunction, the drone can remain airborne and possibly return to a base for repairs, a feature critical in the unforgiving and remote environment of an alien moon. Additionally, the enhanced stability of the hexacopter design, combined with its agility, allows for precise maneuvers, making it adept at both scientific reconnaissance and capturing detailed imagery.

While each drone configuration holds its unique advantages, the primary objective remains to ensure reliable, sustained, and efficient flight in Titan's distinct atmospheric conditions. The Hexacopter, with its balance of power, redundancy, and maneuverability, stands out as the most suitable candidate for this ambitious endeavor.

HEXACOPTER DYNAMIC MODELLING

The dynamics of a hexacopter, like other multi-rotor platforms, are derived from the interaction of its rotors with the surrounding environment. The hexacopter possesses six rotors, usually arranged symmetrically in a planar configuration. This arrangement affects the drone's ability to generate forces and moments to control its position and orientation.

Assumptions:

For simplification, the following assumptions are made:

- The hexacopter is a rigid body.
- The rotors produce thrust linearly proportional to the square of their speed.
- All six propellers of the TitanWing Drone are placed orthogonally along with the body frame.

Coordinate Frames:

Two primary coordinate frames are defined to describe the hexacopter motion:

- **Inertial Frame** (R_I): Fixed to the Earth and generally considered as the reference frame.
- **Body Frame** (R_B): Fixed to the center of mass of the hexacopter, with axes aligned to the drone's principal axes, found in Figure 1.

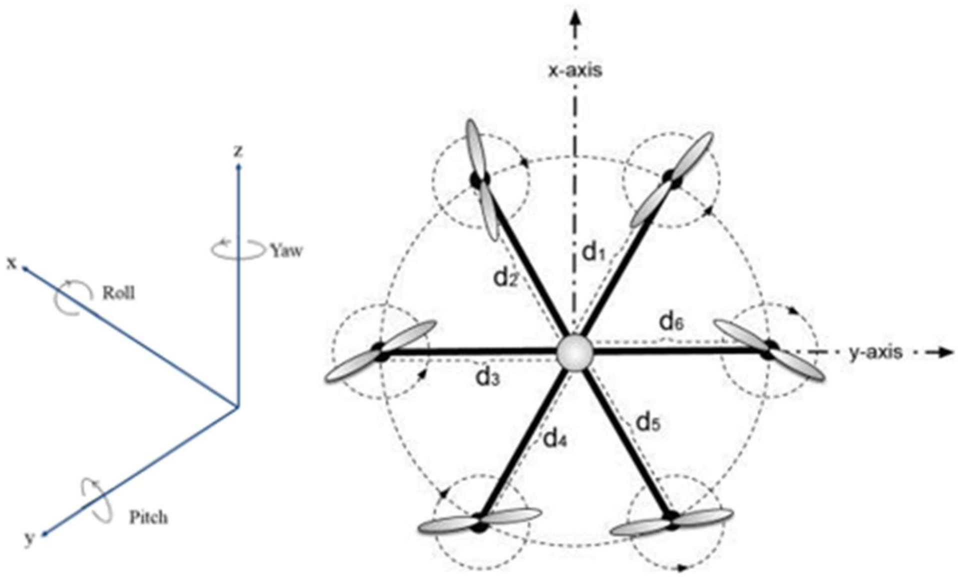

Figure 1. The schematic structure of the hexacopter and rotational directions of the propellers.

Rotation Matrix R

A rigid spacecraft can be described as a system of particles where the relative distances are fixed during time. Figure 2 is an illustration of the drone with an inertial (R_I) and body (R_B) frames shown. The $X, Y \ and \ Z$ axes represent the $\vec{\imath}$, $\vec{\jmath}$ and $\vec{k}$ axes respectively. The center of the inertial reference frame $\bar{I}$ is located at Titan's CM. The $\vec{k}_{\bar{I}}$ axis is along the earth's axis of rotation while the $\vec{\imath}_{\bar{I}}$ axis points toward the vernal equinox and the $\vec{\jmath}_{\bar{I}}$ axis completes the right-handed frame. The satellite's body frame, $\bar{B}$, is defined by $\vec{\imath}_{\bar{B}}, \vec{\jmath}_{\bar{B}}$ and $\vec{k}_{\bar{B}}$ axes. The Euler angles are the rotational angles about the body axes defined as followed:

$$\psi \ about \ \vec{k}_{\bar{b}} = [R_z(\psi)]$$
$$\theta \ about \ \vec{\jmath}_{\bar{b}} = [R_y(\theta)]$$
$$\phi \ about \ \vec{\imath}_{\bar{b}} = [R_x(\phi)]$$

This is a 3 → 2 → 1 rotation that will be used to find the direction cosine matrix (DCM).

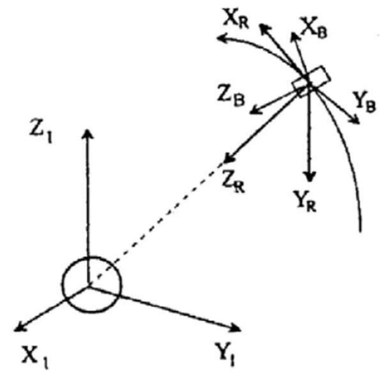

Fig. 1. Inertial (I), orbital reference (R) and body (B) frames.

Figure 2. Inertial reference frame (I), and body frame (B).

The direction cosine matrix, equation (1), can be found by implementing the rotation sequence mentioned above.

$$^{\bar{I}}[c]^{\bar{B}} = [R_x(\phi)]\cdot[R_y(\theta)]\cdot[R_z(\psi)] = \begin{bmatrix}1 & 0 & 0\\ 0 & Cos\phi & Sin\phi\\ 0 & -Sin\phi & Cos\phi\end{bmatrix}\cdot\begin{bmatrix}Cos\theta & 0 & -Sin\theta\\ 0 & 1 & 0\\ Sin\theta & 0 & Cos\theta\end{bmatrix}\cdot\begin{bmatrix}Cos\psi & Sin\psi & 0\\ -Sin\psi & Cos\psi & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$^{\bar{I}}[c]^{\bar{B}} = \begin{bmatrix}\mathrm{Cos}\theta\mathrm{Cos}\psi & \mathrm{Cos}\theta\mathrm{Sin}\psi & -\mathrm{Sin}\theta\\ \mathrm{Cos}\psi\mathrm{Sin}\theta\mathrm{Sin}\phi-\mathrm{Cos}\phi\mathrm{Sin}\psi & \mathrm{Cos}\phi\mathrm{Cos}\psi+\mathrm{Sin}\theta\mathrm{Sin}\phi\mathrm{Sin}\psi & \mathrm{Cos}\theta\mathrm{Sin}\phi\\ \mathrm{Cos}\phi\mathrm{Cos}\psi\mathrm{Sin}\theta+\mathrm{Sin}\phi\mathrm{Sin}\psi & -\mathrm{Cos}\psi\mathrm{Sin}\phi+\mathrm{Cos}\phi\mathrm{Sin}\theta\mathrm{Sin}\psi & \mathrm{Cos}\theta\mathrm{Cos}\phi\end{bmatrix} \quad (1)$$

The Rotation Matrix R_B^I which is used to transform the body frame in terms of the inertial frame is given by equation (2).

$$R_B^I = \begin{bmatrix}\mathrm{Cos}\theta\mathrm{Cos}\psi & \mathrm{Cos}\psi\mathrm{Sin}\theta\mathrm{Sin}\phi-\mathrm{Cos}\phi\mathrm{Sin}\psi & \mathrm{Cos}\phi\mathrm{Cos}\psi\mathrm{Sin}\theta+\mathrm{Sin}\phi\mathrm{Sin}\psi\\ \mathrm{Cos}\theta\mathrm{Sin}\psi & \mathrm{Cos}\phi\mathrm{Cos}\psi+\mathrm{Sin}\theta\mathrm{Sin}\phi\mathrm{Sin}\psi & -\mathrm{Cos}\psi\mathrm{Sin}\phi+\mathrm{Cos}\phi\mathrm{Sin}\theta\mathrm{Sin}\psi\\ -\mathrm{Sin}\theta & \mathrm{Cos}\theta\mathrm{Sin}\phi & \mathrm{Cos}\theta\mathrm{Cos}\phi\end{bmatrix} \quad (2)$$

The orientation vector $\eta = [\ \phi\ \theta\ \psi\ ]^T$ is formed using the three Euler angles, yaw angle ψ, pitch angle θ, and roll angle ϕ. The vector $\xi = [x\ y\ z]^T$ will denote the position of the vehicle in the inertial reference frame.

Rigid Body Kinematics

<u>Hexacopter Kinematics</u>

To find the relationship between the position and velocities of the drone relative to the inertial reference frame, the Rotation Matrix R_B^I can be used as shown in equation (3).

$$\frac{d}{dt}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \dot{R}_B^I\begin{bmatrix}u\\ v\\ w\end{bmatrix} = \begin{bmatrix}\mathrm{Cos}\theta\mathrm{Cos}\psi & \mathrm{Cos}\psi\mathrm{Sin}\theta\mathrm{Sin}\phi-\mathrm{Cos}\phi\mathrm{Sin}\psi & \mathrm{Cos}\phi\mathrm{Cos}\psi\mathrm{Sin}\theta+\mathrm{Sin}\phi\mathrm{Sin}\psi\\ \mathrm{Cos}\theta\mathrm{Sin}\psi & \mathrm{Cos}\phi\mathrm{Cos}\psi+\mathrm{Sin}\theta\mathrm{Sin}\phi\mathrm{Sin}\psi & -\mathrm{Cos}\psi\mathrm{Sin}\phi+\mathrm{Cos}\phi\mathrm{Sin}\theta\mathrm{Sin}\psi\\ -\mathrm{Sin}\theta & \mathrm{Cos}\theta\mathrm{Sin}\phi & \mathrm{Cos}\theta\mathrm{Cos}\phi\end{bmatrix}\begin{bmatrix}u\\ v\\ w\end{bmatrix} \quad (3)$$

Similarly, we can find the relationship between Euler angles and the angular rates p, q, and r as follows (4):

$$\omega = R_r\dot{\eta} = \begin{bmatrix}p\\ q\\ r\end{bmatrix} = \begin{bmatrix}1 & 0 & -\mathrm{Sin}\theta\\ 0 & \mathrm{Cos}\phi & \mathrm{Sin}\phi\mathrm{Cos}\theta\\ 0 & -\mathrm{Sin}\phi & \mathrm{Cos}\theta\mathrm{Cos}\phi\end{bmatrix}\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} \quad (4)$$

Rigid Body Dynamics

We will define the velocity of the hexacopter as V_i and apply Newton's second law $f = ma = m\frac{dV_i}{dt}$. Here $m = mass$, $f = net\ force$, and $\frac{dV_i}{dt} = time\ derivative$ in the inertial reference frame. The given equations for the translational equations of motion for an aircraft or a similar rigid body in a three-dimensional space need to be described. They describe the linear accelerations $(\dot{u}, \dot{v}, \dot{w})$ in the body-fixed coordinate system, which is typically defined as follows:

- u, v, and w are the linear velocities along the body x (longitudinal), y (lateral), and z (vertical) axes, respectively.
- p, q, and r are the angular velocities (roll, pitch, and yaw rates, respectively) about the body x, y, and z axes, respectively.
- f_x, f_y, and f_z are the forces acting on the body along the x, y, and z axes, respectively.
- m is the mass of the body/drone.

The equations can be written as follows (5):

$$\begin{aligned} \dot{u} &= rv - qw + \frac{f_x}{m} \\ \dot{v} &= pw - ru + \frac{f_y}{m} \\ \dot{w} &= qu - pv + \frac{f_z}{m} \end{aligned} \tag{5}$$

The first part of each equation (e.g., $rv - qw$) represents the inertial forces due to the rotation of the body. The second part $(1/m)[fx, fy, fz]$ represents the acceleration due to the net external force acting on the body in the respective direction, divided by the mass of the body. This formulation is derived from Newton's second law of motion, which states that the acceleration of a body is directly proportional to the net force acting upon the body and inversely proportional to the body's mass.

These equations are crucial in flight dynamics to understand and predict the motion of an aircraft or other aerial vehicles under various force and moment conditions.

As noted, assuming the system of particles is a rigid body with body frame $\bar{B}$, and the point B at the origin of the rigid body is assumed to be the CM, i.e., B = CM, then the torque of the rigid body about point B can be written as shown in equation (6).

$$\vec{\tau}_B = {}^{\bar{I}}\frac{d}{dt}\, {}^{\bar{I}}_{I}\vec{h}_B \tag{6}$$

Here ${}^{\bar{I}}_{I}\vec{h}_B$ is the angular momentum of the system about point B with respect to the inertial reference frame $\bar{I}$. Since point B is assumed to be the CM of the system, the angular momentum can be written as follows in equation (2) where $\tilde{I}_B$ is the tensor of inertia about point B and ${}^{\bar{I}}\omega^{\bar{B}}$ is the angular velocity of the body with respect to the inertial reference frame $\bar{I}$. Note that mass moment of inertia terms is not assumed to be principal body axes (7).

$${}^{\bar{I}}_{I}\vec{h}_B = \tilde{I}_B \cdot {}^{\bar{I}}\omega^{\bar{B}} \tag{7}$$

$$\tilde{I}_B = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix}$$

$$^{\bar{I}}\omega^{\bar{B}} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

However, the hexacopter is completely symmetric about all three axes, therefore $I_{xx} = I_{yy} = I_{zz} = 0$. The inertia tensor, $\tilde{I}_B$, can be rewritten as shown in equation (8).

$$\tilde{I}_B = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \quad (8)$$

Applied Forces and Torques

The model has been enhanced to be more true-to-life by incorporating an analysis of air resistance and rotor drag, in addition to considering the gravitational and thrust forces from the rotor. The movements of the unmanned aerial vehicle (UAV) are dictated by either aerodynamic or mechanical influences, adding to the complexity of the UAV. To derive the mathematical model of the hexacopter, Newtonian mechanics is employed, resulting in the subsequent equations. The Newton-Euler equations integrate Newton's second law of motion and Euler's rotational equations of motion, describing the linear and angular dynamics of a rigid body. Mathematically, these are expressed as:

$$\begin{bmatrix} mI_{3x3} & 0_{3x3} \\ 0_{3x3} & \tilde{I}_B \end{bmatrix} \begin{bmatrix} \dot{V} \\ \dot{\omega} \end{bmatrix} + \begin{bmatrix} \omega\Lambda(mV) \\ \omega\Lambda(\tilde{I}_B\omega) \end{bmatrix} = \begin{bmatrix} \sum F \\ \sum \tau \end{bmatrix} \quad (9)$$

Where:

- m is the mass of the body.
- I_{3x3} is the 3 x 3 identity matrix.
- 0_{3x3} is a 3 x 3 zero matrix.
- $\tilde{I}_B$ is the inertia tensor matrix.
- V and ω are the linear and angular velocity vectors, respectively.
- $\Lambda(\omega)$ is the skew-symmetric matrix formulated from ω.
- $\dot{V}$ and $\dot{\omega}$ are the linear and angular accelerations, respectively.
- $\sum F$ and $\sum \tau$ are the resultant external force and moment (torque) acting on the body, respectively.
- **Linear and Angular Velocities and Accelerations** ($\boldsymbol{V}, \boldsymbol{\omega}, \dot{V}, \dot{\omega}$): These quantities describe the instantaneous rate of change of position and orientation of the rigid body.
- **Mass and Inertia Tensor** ($\boldsymbol{m}, \tilde{I}_B$)**:** These scalar and matrix terms, respectively, account for the distribution of mass in the body, influencing its resistance to translational and rotational accelerations.
- **Skew-symmetric Matrix** ($\boldsymbol{\Lambda(\omega)}$): This matrix, derived from the angular velocity vector, facilitates the representation of cross products in matrix multiplication form.
- **Resultant External Force and Moment** ($\sum\boldsymbol{F}, \sum\boldsymbol{\tau}$): These represent the total external influences acting on the body, inducing translational and rotational motion.

We can further break down equation (9) into the following set of equations (10) & (11):

$$\sum F_B = m\dot{V} + \omega \times mV \quad (10)$$

$$\sum \tau = \tilde{I}_B \dot{\omega} + \omega \times \tilde{I}_B \omega \quad (11)$$

Forces

Gravitational Force Analysis

In the context of hexacopter dynamics, the gravitational force, fundamentally directed towards the earth's/Titan's center, imparts a pivotal influence on the system, especially when analyzed in the body-fixed coordinate frame. The vector denoting gravitational force (F_g), equation (12), acting upon the hexacopter center of gravity can be articulated as:

$$F_g = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} \quad (12)$$

Here, m symbolizes the hexacopter mass, and g represents the gravitational acceleration, conventionally directed negatively along the body's z-axis in the body coordinate frame. This formulation succinctly encapsulates the gravitational force's impact on the hexacopter, providing a fundamental building block for further dynamic analyses and control design in aerial robotics.

Thrust Force Analyses

In a hexacopter, the propeller thrust dynamics are intricately linked with the rotational speeds of the propellers and certain aerodynamic coefficients. The thrust force generated by each propeller can be meticulously expressed by considering the propeller's individual characteristics and rotational speeds. Given this, the thrust force vector F_p, equation (13), in the body frame can be articulated as:

$$F_i = (\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2 + \Omega_5^2 + \Omega_6^2)$$

$$F_P = R_B^I \begin{bmatrix} 0 \\ 0 \\ \sum_{i=1}^{6} F_i \end{bmatrix} = R_B^I [\sum_{i=1}^{6} b\Omega_i^2] \quad (13)$$

where:

- F_p is the total propeller thrust force vector in the inertial frame.
- R_B^I is the rotation matrix that transforms vectors from the body frame to the inertial frame.
- F_i represents the thrust produced by the ith propeller, with i ranging from 1 to 6, encompassing all propellers of the hexacopter.
- b is the thrust coefficient, a factor that relates the square of the propeller rotational speed (Ω_i) to the generated thrust.
- Ω_i is the rotational speed of the ith propeller.

In this formulation, $\sum_{i=1}^{6} F_i$ or $\sum_{i=1}^{6} b\Omega_i^2$ computes the total thrust generated by all propellers, which is then oriented in the inertial frame through multiplication with R_B^I. This detailed representation is essential for accurate modeling and control of hexacopter, providing a mechanism to understand how variations in propeller speeds influence the total thrust and, consequently, the drone's motion. This approach allows for an enriched understanding and control mechanism design by correlating propeller actuation with the resultant force generation and vehicle dynamics.

Translational Drag Force

The translational drag force F_t pertains to the resistive forces experienced by the hexacopter as it translates through the air. This drag force is influenced by the hexacopter's linear velocity and is typically modeled to be proportional to the velocity (or sometimes the square of the velocity) in each axis. This force does not directly influence the yaw motion but impacts the translational dynamics of the hexacopter, affecting its motion along the x, y, and z axes.

The translational drag force, equation (14), is expressed as follows:

$$F_t = k_{ft} \cdot V \tag{14}$$

Where:

- F_t is the translational drag force vector.
- k_{ft} is a diagonal matrix representing drag coefficients along the x, y, and z axes, defined as: $k_{ft} = \begin{bmatrix} k_{ftx} & 0 & 0 \\ 0 & k_{fty} & 0 \\ 0 & 0 & k_{ftz} \end{bmatrix}$
- V is the linear velocity vector of the hexacopter, obtained as the derivative of the position vector, i.e., $V = \dot{\xi}$, with $\xi = [x, y, z]^T$ representing the position.

The drag force is derived based on empirical observations and fluid dynamics principles, observing that the force exerted by a fluid on a moving object is proportional to the velocity of the object. Specifically,

$$F_{t_i} = -k_{ft_i} V_i \tag{15}$$

for each axis $i \in \{x, y, z\}$. The negative sign indicates that the drag force opposes the motion.

Rotor Drag Torque

The rotor drag torque, τ_D, is the torque induced due to the rotation of the propellers, which tends to generate a yawing motion (rotation about the vertical axis) of the hexacopter. In typical multi-rotor configurations, propellers are arranged such that adjacent ones rotate in opposite directions to minimize the net yawing moment.

The rotor drag torque is derived by considering Newton's third law and the principle that the angular momentum imparted to the air by the propeller is equal and opposite to that imparted to the hexacopter. The individual rotor drag torque is modeled to be proportional to the square of the rotational speed, i.e.,

$$\tau_{D_i} = CA\rho r^2 \Omega_i^2 = k_d \Omega_i^2 \tag{16}$$

Where, $C = drag\ coefficient$ of the propeller, $A = area\ of\ the\ blade$, $\rho = air\ density$, $r = blade\ radius$ and $\Omega_i = propeller\ angular\ velocity$. Summing these torques while considering the opposite rotation directions of alternate propellers, we obtain the total rotor drag torque, τ_D.

The total rotor drag torque, equation (17), can be expressed as:

$$\tau_D = \begin{bmatrix} 0 \\ 0 \\ \sum_{i=1}^{6} (-1)^i \tau_{D_i} \end{bmatrix} \tag{17}$$

Here:

- τ_{D_i} is the drag torque induced by the ith propeller.
- k_d is the drag coefficient, capturing the proportionality between the square of the propeller rotational speed and the generated drag torque.
- Ω_i is the rotational speed of the ith propeller.
- τ_D is the total drag torque vector acting about the body axes of the hexacopter.
- The summation $\sum_{i=1}^{6}(-1)^i\tau_{D_i}$accounts for all propellers, with the alternating sign $(-1)^\wedge i$ accounting for the opposing rotation direction of alternate propellers (assuming a configuration where neighboring propellers rotate in opposite directions).

In a typical hexacopter configuration, propellers are arranged such that consecutive propellers rotate in opposite directions to counteract the yaw moment induced by each other. This arrangement minimizes the net yaw moment on the hexacopter, providing yaw stability and control.

Torques

In the context of a hexacopter, actuator actions primarily pertain to the thrusts generated by the six propellers. These thrusts, apart from inducing translational motion, also generate torques (moments) about the three principal axes of the hexacopter due to their application points being offset from the center of mass.

1. Roll Torque (M_x)

   Roll torque is generated due to the differential thrust produced by propellers that are positioned symmetrically about the roll axis $(x - axis)$. For calculating the roll torque M_x, which is the torque about the $x - axis$, we need to consider the component of the lever arm that is perpendicular to the $x - axis$ (i.e., in the $y - z$ plane). If a propeller is located at an angle θ relative to the $x - axis$ in the $y - z$ plane, then the effective lever arm length for calculating M_x is:

$$l_{eff} = lsin(\theta)$$

   Given that the hexacopter is symmetric and propellers are equally spaced, the propellers that have a component of their position in the $y - z$ plane will be at an angle of 30° to the $z - axis$ (assuming propellers are numbered sequentially around the hexacopter). Thus, $\theta = 30°$ and:

$$l_{eff} = lsin(30°) = \frac{l}{2}$$

   Considering a typical hexacopter configuration as mentioned above, the roll torque M_x, equation (18), can be derived as:

$$M_x = \frac{-lT_1 - lT_2 - lT_3 + lT_4 + lT_5 + lT_6}{2} \quad (18)$$

   Here:
- l is the distance from the propeller to the center of gravity along the y-axis.
- T_i is the thrust produced by the ith propeller.

   The negative or positive signs and factors in front of each term depend on the configuration and rotation direction of each propeller. They ensure that a positive roll torque (according to the right-hand rule) corresponds to a roll motion to the right (clockwise when viewed from behind).

2. Pitch Torque (M_y)

Similarly, pitch torque is due to the differential thrust produced by propellers positioned symmetrically about the pitch axis ($y - axis$). In a common hexacopter configuration, the six propellers are equally spaced around the circumference of a circle, i.e., each propeller is 60° apart from its neighbors. This configuration often involves an alternating pattern of propeller rotation directions.

The pitch torque M_y is influenced by propellers that have a lever arm component along the $y - axis$. If we consider a coordinate system where the $x - axis$ is pointing forward and the $y - axis$ is pointing to the right (following the right-hand rule for a $Z - up$ coordinate system, see Figure 2), propellers 1 and 4 are located along the x-axis and thus do not contribute to M_y due to their lever arm being zero in the y-direction.

The key contributors to M_y are propellers located along or having a component along the y-axis. If we take propeller 3 as an example, which is typically located at a 60° angle to the $x - axis$ (in the $-y$ direction in our coordinate system), the lever arm length in the y-direction is:

$$l_y = lsin(60°)$$

Since$\sin(60°) = \sqrt{3}/2$, we introduce a factor of $\sqrt{3}$ into our torque expression. The total moment arm in the y-direction for each propeller (depending on its position) is used in the computation of M_y. Given this understanding, the expression for M_y, equation (19), can be derived as:

$$M_y = \frac{\sqrt{3}lT_1 - \sqrt{3}lT_3 - \sqrt{3}lT_4 + \sqrt{3}lT_6}{2} \quad (19)$$

Here:

- l is the distance from the propeller to the center of gravity along the x-axis.

The coefficients and signs are derived based on the hexacopter configuration, ensuring a positive pitch torque results in a pitch-down motion.

3. Yaw Torque (M_z)

Yaw torque is slightly different as it arises due to the aerodynamic torque produced by each propeller, which is related to its rotational speed and direction. The yaw torque M_z is the net torque about the $z - axis$ (vertical axis) of the hexarotor. It's influenced by the aerodynamic torques produced by each propeller, which arise due to the motor exerting a torque to spin the propeller and, by Newton's third law, the propeller exerting an equal and opposite torque back on the motor (and thus, the hexacopter).

- **Aerodynamic Torque:** Each propeller generates aerodynamic torque τ_i due to its rotation.
- **Rotation Directions:** The sign of each term in the expression for M_z is determined by the direction of rotation of each propeller. In a common hexacopter configuration, adjacent propellers rotate in opposite directions to help counteract the yaw torques they produce.
- **Net Yaw Torque:** The net yaw torque M_z, equation (20), is the sum of the aerodynamic torques from all the propellers, considering their rotation directions:

$$M_z = \sum_{i=1}^{6} (-1)^i \tau_i$$
$$M_z = -\tau_1 + \tau_2 - \tau_3 + \tau_4 - \tau_5 + \tau_6 \quad (20)$$

Here:

- τ_i is the aerodynamic torque produced by the ith propeller.

The signs ensure that a positive yaw torque corresponds to a clockwise yaw motion when viewed from above, consistent with the right-hand rule.

Propeller Thrust and Torque

The thrust T_i and aerodynamic torque τ_i produced by a propeller i are related to its rotational speed Ω_i as follows, equations (21 & 22):

$$T_i = b\Omega_i^2 \quad (21)$$

$$\tau_i = d\Omega_i^2 \quad (22)$$

With:

- $b = C_T \rho r_p^4 \pi$ (Thrust factor)
- $d = C_Q \rho r_p^5 \pi$ (Torque factor)
- C_T and C_Q are the thrust and torque coefficients respectively.
- ρ is the air density.
- r_p is the propeller radius.

Torque Vector (M_f)

The vector of the torques produced by the hexacopter about its principal axes is given by equation (23):

$$M_f = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} \quad (23)$$

Aerodynamic Resistance Torque

In aerial vehicles like a hexacopter, aerodynamic resistance can introduce torques that act against the rotation of the propellers. This phenomenon is sometimes referred to as "drag torque" or "aerodynamic torque," which is a force that resists the motion of the propellers through the air.

General Form:

The torque due to aerodynamic resistance τ_{aero} for a single propeller might be expressed as:

$$\tau_{aero,i} = k_w \Omega_i$$

or,

$$\tau_{aero,i} = k_w \Omega_i^2$$

depending on the specific aerodynamic model used.

Here:

- Ω_i is the angular velocity of the propeller.
- k_w is a coefficient that represents the proportionality between the angular velocity of the propeller and the induced aerodynamic torque.

When considering a hexacopter, each propeller will generate its own aerodynamic torque. The net aerodynamic torque τ_{aero} acting on the hexacopter would then be the sum of the aerodynamic torques from all the propellers.

$$\mathrm{T}_{aero} = \sum_{i=1}^{6} \tau_{aero,i}$$

Considering a hexacopter subjected to aerodynamic resistance torques during rotational motion, a model is proposed:

$$\tau_{aero} = M_a = \begin{bmatrix} k_{fa_x}(\dot{\phi})^2 \\ k_{fa_y}(\dot{\theta})^2 \\ k_{fa_z}(\dot{\psi})^2 \end{bmatrix} \tag{24}$$

Where:

- M_a is the aerodynamic resistance torque vector.
- $k_{fa_x}, k_{fa_y}, k_{fa_z}$: Aerodynamic constants along $x, y, and\ z\ axes$, respectively.
- $\dot{\phi}$, $\dot{\theta}$, $\dot{\theta}$: Angular velocities about $x\ (roll), y\ (pitch), and\ z\ (yaw)\ axes$, respectively.
- $k_{fa_x}(\dot{\phi})^2$: Torque opposing roll, proportional to the square of roll rate.
- $k_{fa_y}(\dot{\theta})^2$: Torque opposing pitch, proportional to the square of pitch rate.
- $k_{fa_z}(\dot{\psi})^2$: Torque opposing yaw, proportional to the square of yaw rate.

<u>Gyroscopic Effects</u>

The gyroscopic effect generated by the rotating propellers of a hexacopter (or any rotorcraft) can indeed have a significant impact on its dynamics, especially during maneuvers involving changes in orientation. The gyroscopic effect arises due to the precession of the rotating propellers, which can be expressed as equation (25) (assuming a coordinate system where the z-axis is vertical):

$$M_g = \tilde{I}_{rotor} \cdot \left(\omega_{body} \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right) \Omega_{rotor}$$

$$M_g = \tilde{I}_{rotor} \cdot \left(\begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right) \Omega_{rotor} = \tilde{I}_{rotor} \begin{bmatrix} \dot{\theta} \\ \dot{\phi} \\ 0 \end{bmatrix} \Omega_{rotor} \tag{25}$$

Where:

- $\tilde{I}_{rotor}$ is the rotor inertia.
- $\omega_{body} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$ is the angular velocity of the hexarotor in the body frame.
- Ω_{rotor} is the net rotor speed, which is the sum of the angular speeds of the individual rotors, considering their rotation directions.
- Rotor Speeds: $\Omega_{rotor} = -\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4 - \Omega_5 + \Omega_6$ considers the rotation directions of each rotor. Adjacent rotors typically rotate in opposite directions to cancel out the yaw torques they produce.
- Cross Product: The cross product $\omega_{body} \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} q \\ -p \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ \dot{\phi} \\ 0 \end{bmatrix}$ gives the perpendicular vector to ω and the $z - axis$, which is consistent with the gyroscopic torque being perpendicular to the axis of rotation and the angular momentum vector.

Mathematical Model

<u>Translational Dynamics</u>

The following mathematical model describes the translational dynamics of the hexacopter in three-dimensional space, considering various forces such as thrust, gravitational, and translational drag. Let's go through it in detail:

The governing equation:

$$m\ddot{\xi} = F_p + F_g - F_t = \sum F \tag{26}$$

Combining equations (12), (13), and (14), we get the following translational dynamic equations (27), (28), and (29):

$$\ddot{x} = \frac{(cos\phi cos\psi sin\theta + sin\phi sin\psi)\sum_{i=1}^{6} F_i - k_{ftx}\dot{x}}{m} \tag{27}$$

$$\ddot{y} = \frac{(cos\phi sin\theta sin\psi - sin\phi cos\psi)\sum_{i=1}^{6} F_i - k_{fty}\dot{y}}{m} \tag{28}$$

$$\ddot{z} = \frac{(cos\phi cos\theta)\sum_{i=1}^{6} F_i - k_{ftz}\dot{z}}{m} - g \tag{29}$$

Where:

- $(\ddot{x}, \ddot{y}, \ddot{z})$ are the linear accelerations along the x, y, and z-axes respectively.
- (ϕ, θ, ψ) are the roll, pitch, and yaw angles respectively.

- F_i is the thrust produced by the ith propeller.
- $k_{ftx}, k_{fty}, k_{ftz}$ are the translational drag coefficients along the $x, y, and\ z - axes$ respectively,
- g is the gravitational acceleration.

Rotational Dynamics

The following mathematical model describes the translational dynamics of the hexacopter in three-dimensional space, considering various torques such as those due to motor thrust, aerodynamic drag, and gyroscopic effects.

The governing equation:

$$\tilde{I}_B \dot{\omega} = -\omega \times \tilde{I}_B \omega + M_f - M_a - M_g = \sum M \tag{30}$$

Combining equations (18), (19), (20), (24), and (25) we get the following rotational dynamic equations (31), (32), and (33):

1. **Roll (Around x-axis):**

$$I_{xx}\ddot{\phi} = \dot{\theta}\dot{\psi}\left(I_{yy} - I_{zz}\right) - k_{fax}\left(\dot{\phi}\right)^2 - I_r\Omega_r\dot{\theta} + bl(-\Omega_2^2 + \Omega_5^2 + \frac{-\Omega_1^2 - \Omega_3^2 + \Omega_4^2 + \Omega_6^2}{2} \tag{31}$$

2. **Pitch (Around y-axis):**

$$I_{yy}\ddot{\theta} = \dot{\phi}\dot{\psi}(I_{zz} - I_{xx}) - k_{fay}\left(\dot{\theta}\right)^2 + I_r\Omega_r\dot{\phi} + bl(\frac{\sqrt{3}(-\Omega_1^2 + \Omega_3^2 + \Omega_4^2 + \Omega_6^2)}{2} \tag{32}$$

3. **Yaw (Around z-axis):**

$$I_{zz}\ddot{\psi} = \dot{\phi}\dot{\theta}\left(I_{xx} - I_{yy}\right) - k_{faz}\left(\dot{\psi}\right)^2 + d(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2 - \Omega_5^2 + \Omega_6^2) \tag{33}$$

The control input vector $\boldsymbol{U}_T = [u_1, u_2, u_3, u_4]^T$ typically represents a set of generalized control inputs, which might be defined to represent collective thrust and torques about the roll, pitch, and yaw axes, respectively, for a hexacopter control system. A general way of relating them to the motor speeds in a hexacopter can be done as followed:

$$U_T = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \mathrm{A} \cdot \Omega^2 \tag{34}$$

Where:

- $\boldsymbol{U}_T = [u_1, u_2, u_3, u_4]^T$ is the control input vector, typically representing total thrust, roll torque, pitch torque, and yaw torque respectively.
- A is a mixing matrix that relates the square of the motor speeds to the control inputs.
- $\boldsymbol{\Omega} = [\Omega_1, \Omega_2, \Omega_3, \Omega_4, \Omega_5, \Omega_6]^T$ is a vector of the rotor speeds.

The mixing matrix A relates the square of the motor speeds to the generated thrust and torques and depends on the hexacopter's configuration (i.e., the position and orientation of each rotor). It is defined as:

$$A = \begin{bmatrix} b & b & b & b & b & b \\ -\frac{bl}{2} & -bl & -\frac{bl}{2} & \frac{bl}{2} & bl & \frac{bl}{2} \\ -\frac{bl\sqrt{3}}{2} & 0 & \frac{bl\sqrt{3}}{2} & \frac{bl\sqrt{3}}{2} & 0 & -\frac{bl\sqrt{3}}{2} \\ -d & d & -d & d & -d & d \end{bmatrix}$$

Where:

- b is a thrust coefficient.
- l is the distance from each rotor to the center of mass.
- d is a drag coefficient.

Explanation:

- u_1 (Total Thrust): All rotors contribute equally to total thrust when spinning at the same speed.
- u_2 (Roll Torque): Rotors on opposite sides of the roll axis contribute oppositely to roll torque.
- u_3 (Pitch Torque): Rotors on opposite sides of the pitch axis contribute oppositely to pitch torque.
- u_4 (Yaw Torque): Rotors spinning in opposite directions contribute oppositely to yaw torque.

Therefore, our control input vector U_T can be represented by the following equation (35):

$$U_T = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \mathrm{A} \cdot \Omega^2 = A = \begin{bmatrix} b & b & b & b & b & b \\ -\frac{bl}{2} & -bl & -\frac{bl}{2} & \frac{bl}{2} & bl & \frac{bl}{2} \\ -\frac{bl\sqrt{3}}{2} & 0 & \frac{bl\sqrt{3}}{2} & \frac{bl\sqrt{3}}{2} & 0 & -\frac{bl\sqrt{3}}{2} \\ -d & d & -d & d & -d & d \end{bmatrix} \begin{bmatrix} \Omega_1^2 \\ \Omega_2^2 \\ \Omega_3^2 \\ \Omega_4^2 \\ \Omega_5^2 \\ \Omega_6^2 \end{bmatrix} \tag{35}$$

Applying the model dynamics, 2nd order differential equations for the hexacopter's position and orientation in space can be obtained and expressed as shown in equations (36-40):

$$\ddot{\phi} = \frac{[\dot{\theta}\dot{\psi}(I_{yy}-I_{zz}) - k_{fax}\dot{\phi}^2 - I_r\Omega_r\dot{\theta} + u_2]}{I_{xx}} \tag{36}$$

$$\ddot{\theta} = \frac{[\dot{\phi}\dot{\psi}(I_{zz}-I_{xx}) - k_{fay}\dot{\theta}^2 + I_r\Omega_r\dot{\phi} + u_3]}{I_{yy}} \tag{36}$$

$$\ddot{\psi} = \frac{[\dot{\phi}\dot{\theta}(I_{xx}-I_{yy}) - k_{faz}\dot{\psi}^2 + u_4]}{I_{zz}} \tag{36}$$

$$\ddot{x} = \frac{[-k_{ftx}\dot{x} + u_x u_1]}{m} \tag{36}$$

$$\ddot{y} = \frac{[-k_{fty}\dot{y} + u_y u_1]}{m} \tag{36}$$

$$\ddot{z} = \frac{[-k_{ftz}\dot{z} + cos\theta cos\phi]}{m} - g \tag{36}$$

Where:

- $u_x = cos\phi cos\psi sin\theta + sin\phi sin\psi$
- $u_y = cos\phi sin\theta sin\psi - sin\phi cos\psi$

SIMULATIONS AND RESULTS

Navigating through the enigmatic atmospheres and landscapes of distant celestial bodies, such as Titan, Saturn's largest moon, presents an exhilarating frontier for aerospace research and exploration. Titan, encapsulated in its thick, nitrogen-dense atmosphere, propounds a unique aerodynamic environment, distinctively different from Earth, thereby warranting meticulous research and simulations to decipher its aerial mysteries. In this section, we delve into meticulous simulations, aiming to comprehend and analyze the translational and rotational dynamics of a hexacopter navigating through Titan's exotic atmospheric conditions. Our motive is not merely to conquer the challenges of autonomous flight in an alien environment but also to unravel the secrets hidden within Titan's lakes, dunes, and potentially, its subsurface ocean. Harnessing the mathematical models and control theories, we intend to create a simulation environment that mimics the physical and atmospheric conditions of Titan. This helps us test, validate, and improve control strategies so that the hexacopter can safely and successfully navigate through Titan's atmosphere. This advances our understanding of this mysterious moon and opens new avenues for solar system exploration in the future. We will conclude three different simulations, *Free Fall, Take-Off* and *Forward Motion* Dynamics and analyze the results of the hexacopter's dynamics on Titan and compare them to that on Earth.

STATE SPACE MODEL

In this section, we introduce the state space model equations for our hexacopter, specifically developed for integration with MATLAB. This compact yet powerful representation captures the essential dynamics of the drone, encompassing variables like position, velocity, and orientation, vital for navigating the unique atmospheric conditions of Titan. Tailored for computational efficiency and precision, these equations are designed to seamlessly integrate with MATLAB's robust simulation and control toolboxes, enabling us to simulate, analyze, and optimize the hexacopter's performance in an environment that closely mimics the challenging conditions of Saturn's largest moon. The equations are derived considering the dynamic model previously derived and can be seen in equations below.

State Variables Vector:

$$X = [\phi \quad \dot{\phi} \quad \theta \quad \dot{\theta} \quad \psi \quad \dot{\psi} \quad x \quad \dot{x} \quad y \quad \dot{y} \quad z \quad \dot{z}]$$

State Space Model Equations:

$x_1 = \phi$ $\quad x_2 = \dot{x_1} = \dot{\phi}$ $\quad x_3 = \theta$ $\quad x_4 = \dot{x_3} = \dot{\theta}$ $\quad x_5 = \psi$ $\quad x_6 = \dot{x_5} = \dot{\psi}$ $\quad x_7 = x$
$x_8 = \dot{x_7} = \dot{x}$ $\quad x_9 = y$ $\quad x_{10} = \dot{x_9} = \dot{y}$ $\quad x_{11} = z$ $\quad x_{12} = \dot{x_{11}} = \dot{z}$

$$\dot{x_2} = \ddot{\phi} = a_1 x_4 x_6 + a_2 x_2^2 + a_3 \Omega_r x_4 + b_1 u_2 \quad (37)$$

$$\dot{x_4} = \ddot{\theta} = a_4 x_2 x_6 + a_5 x_4^2 + a_6 \Omega_r x_2 + b_2 u_3 \quad (38)$$

$$\dot{x_6} = \ddot{\psi} = a_7 x_4 x_2 + a_8 x_6^2 + b_3 u_4 \quad (39)$$

$$\dot{x_8} = \ddot{x} = a_9 x_8 + b_4 u_1 u_x \quad (40)$$

$$\dot{x_{10}} = \ddot{y} = a_{10} x_{10} + b_4 u_1 u_y \quad (41)$$

$$\dot{x_{12}} = \ddot{z} = a_{11} x_{12} + \frac{Cos\phi Cos\theta}{m} u_1 - g \quad (42)$$

To facilitate implementation into MatLab, we have defined the following into our model:

$$a_1 = \frac{I_{yy}-I_{zz}}{I_{xx}} \quad a_2 = \frac{-K_{fax}}{I_{xx}} \quad a_3 = \frac{-I_r}{I_{xx}} \quad a_4 = \frac{I_{zz}-I_{xx}}{I_{yy}} \quad a_5 = \frac{-K_{fay}}{I_{yy}} \quad a_6 = \frac{-I_r}{I_{yy}} \quad a_7 = \frac{I_{xx}-I_{yy}}{I_{zz}}$$

$$a_8 = \frac{-K_{faz}}{I_{zz}} \quad a_9 = \frac{-K_{ftx}}{m} \quad a_{10} = \frac{-K_{fty}}{m} \quad a_{11} = \frac{-K_{ftz}}{m} \quad b_1 = \frac{l}{I_{xx}} \quad b_2 = \frac{l}{I_{yy}} \quad b_3 = \frac{l}{I_{zz}}$$

$$b_4 = \frac{1}{m} \quad (43)$$

ENVIRONMENT AND HEXACOPTER PARAMETERS

Prior to presenting the dynamics observed in our simulations, it is pertinent to outline the environmental conditions and hexacopter parameters that were established. These form the basis of our comparative analysis:

- **Environmental Conditions**:
  - **Earth**:
    - Gravitational acceleration: 9.81 m/s^2
    - Atmospheric density: 1.225 kg/m^3
  - **Titan**:
    - Gravitational acceleration: 1.352 m/s^2
    - Atmospheric density: 5.4 kg/m^3
- **Hexacopter Design Parameters**:
  - Mass: 1.8 kg
  - Rotor diameter: 0.4 meters (larger for Titan's atmosphere)
  - Rotor configuration: Symmetrical hexagonal
  - Rotor count: 6
- **Physical and Inertial Properties**:
  - Length, width, height: 1.2 m, 1.2 m, 0.6 m (similar to NASA's Ingenuity)

- Moments of inertia (I_{xx}, I_{yy}, I_{zz}): Estimated based on drone size and mass distribution

FREE FALL DYNAMICS

Initial Conditions for Free-Fall Dynamics Simulation:

- Starting altitude: Set to a predefined value for both Earth and Titan simulations.
- Initial velocities: Zero in all directions, ensuring a true free-fall scenario from a stationary state.
- Angular orientation: Initially level with no roll, pitch, or yaw angles applied.
- Rotational velocities: Zero, indicating no initial angular momentum.

The simulation results depicted in Figure 3 exhibit the dynamics of the hexacopter in a free-fall scenario on both Earth and Titan. In the absence of wind resistance and with gravity as the sole acting force, the hexacopter's X and Y positions remain constant over time, indicating no lateral movement. This stability suggests an initial state of rest in the horizontal plane or perfectly balanced forces. In contrast, the Z position demonstrates a linear descent, a characteristic of free-fall under gravitational pull. The slower descent rate on Titan corresponds with its weaker gravitational field compared to Earth's. Roll angles for both environments remain unchanged, which implies no rotational movement around the longitudinal axis. These conditions mirror a controlled environment where the hexacopter is released with zero initial velocity or angular momentum, allowing for a direct comparison of gravitational effects in the absence of aerodynamic forces such as lift or wind.

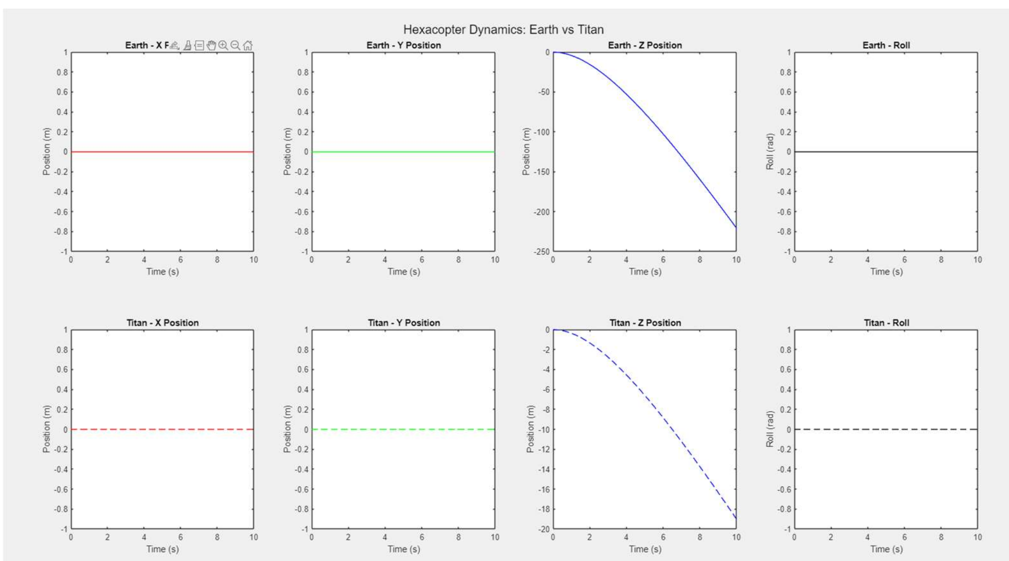

Figure 3. Free Fall Dynamics of TitanWing Earth Vs. Titan

X and Y Position (Earth vs. Titan): The plots for the X and Y positions on Earth and Titan show constant values, indicated by the horizontal lines. This suggests that the hexacopter maintains a constant position in the X and Y directions over time in both environments, which is not entirely realistic unless the hexacopter is designed to hover at a fixed point without any disturbance.

Z Position (Earth vs. Titan): The Z position plots for Earth and Titan show a linear decrease over time, suggesting the hexacopter is falling straight down due to gravity without any lift being generated to counteract it. The rate of descent on Titan appears slower than on Earth, which is consistent with Titan's lower gravity. However, if the hexacopter is supposed to hover or fly, this suggests an issue with the simulation setup where the generated lift is not properly accounted for, or the control inputs are not set to maintain altitude.

Roll (Earth vs. Titan): The roll plots for both environments show a constant value, which might suggest that there's no rolling motion of the hexacopter, or it's perfectly balanced with no disturbance.

TAKE_OFF DYNAMICS

Following our initial analysis of free-fall dynamics, we have now extended our study to include the take-off phase. The additional simulation insights complement the previous findings and provide a comprehensive view of the hexacopter's performance under a takeoff scenario.

Assumptions and Parameters:

- **Design Parameters:** The hexacopter's physical and operational parameters remain consistent with those used in the free-fall simulation. This includes the mass, rotor dimensions, and motor specifications.
- **Initial Conditions:** The initial take-off speed was set at 2 m/s upwards, with no initial velocity in the horizontal axes.
- **Simulation Duration:** A time span of 10 seconds post-take-off was simulated to capture the dynamics adequately.
- **Environmental Settings:** Earth's standard gravity and atmospheric density were compared against Titan's reduced gravity and higher atmospheric density.
- **Aerodynamic Coefficients:** A lift coefficient (C_lift) of 0.8 was applied, which is typical for rotorcraft. The drag coefficient (C_drag) was set at 0.6 after initial simulations indicated numerical stability issues with higher values.

From Figure 4, the simulation revealed a consistent take-off trajectory on both Earth and Titan, with the hexacopter reaching a significantly higher altitude on Titan due to its lower gravity and thicker atmosphere. Implementing a cap on the drag force at 100 N prevented integration errors and allowed the simulation to run without issues. The absence of horizontal motion in the take-off simulation simplified the vertical dynamics, allowing for a focused analysis on ascent performance.

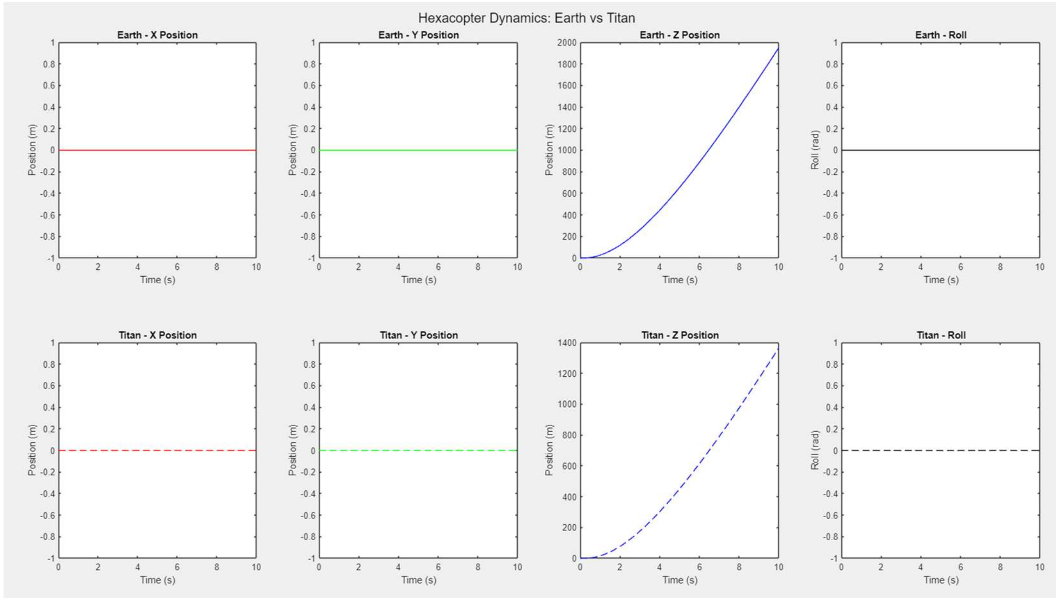

Figure 4. Takeoff Dynamics of TitanWing Earth Vs. Titan

The take-off simulation has affirmed the hexacopter's operational capabilities, exhibiting stable and predictable ascent under the modeled conditions. It also shows the influence of environmental factors on aerodynamic forces, advocating for environment-specific adjustments in design and control strategy. Future work could potentially investigate the refinement of the drag force model and correlate those findings with empirical data.

FORWARD MOTION DYNAMICS

In this section, we present the simulation results for the hexacopter's forward movement dynamics without active stabilization systems. The simulation parameters were set to reflect the initial conditions of a forward speed of 1 m/s and an upward take-off speed of 2 m/s. The forward thrust was incremented to 10 N to analyze the natural response of the hexacopter under these conditions. The results can be seen in Figure 5 below.

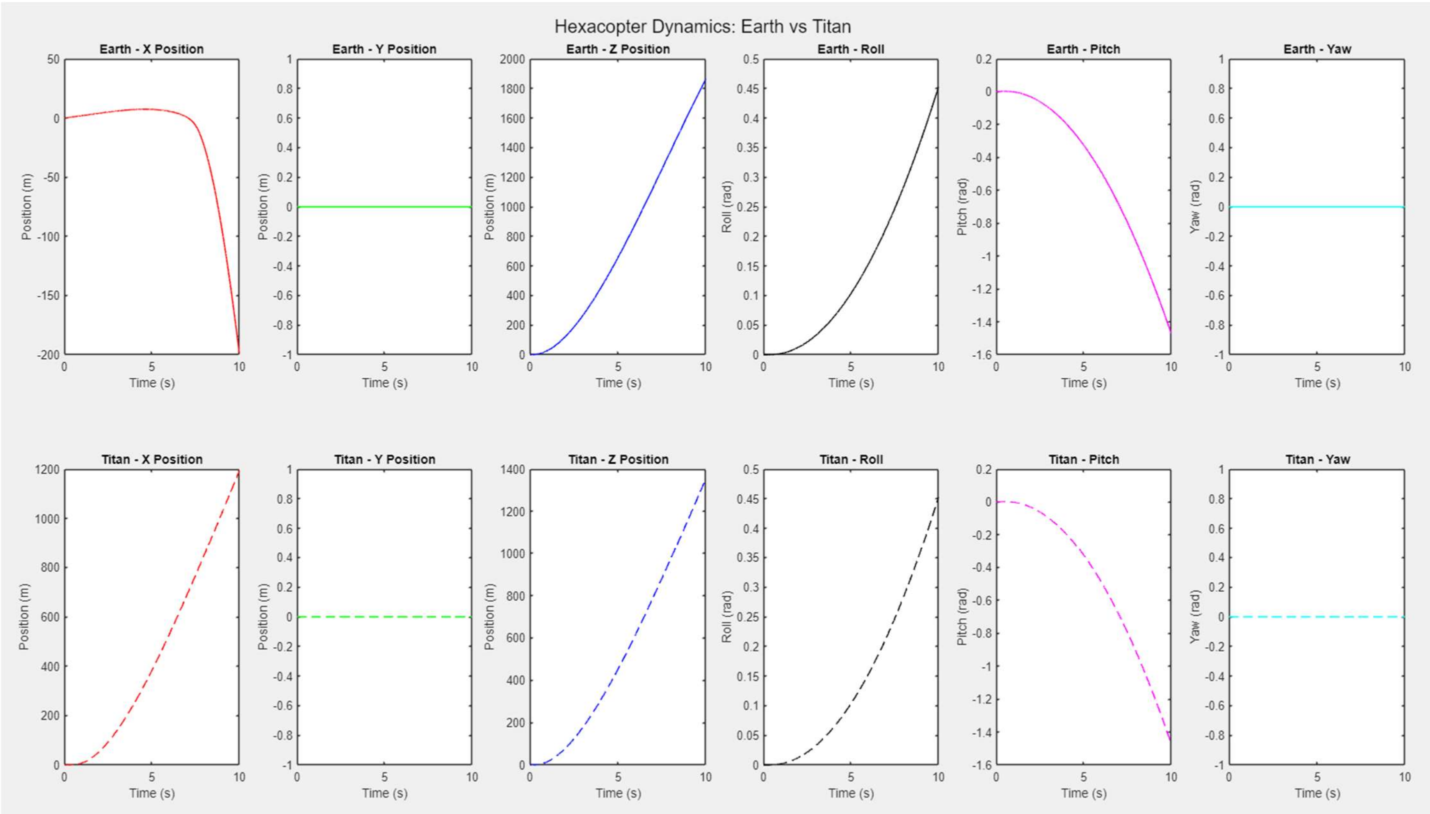

Figure 5. Forward Motion Dynamics of TitanWing Earth Vs. Titan

The following observations were made from the simulation results:

X Position (Forward Movement):

On Earth, the hexacopter initially moves forward but then reverses direction. This could indicate that the forward thrust is insufficient to overcome gravitational pull when combined with the pitch angle, or it may suggest an issue with the thrust direction or aerodynamic drag being modeled.

On Titan, the hexacopter accelerates forward continuously, which is expected due to Titan's lower gravity and potentially its thicker atmosphere (assuming the drag coefficient is set appropriately for Titan's conditions).

Y Position (Lateral Movement):

Both Earth and Titan simulations show no lateral movement as expected, since there are no forces pushing it sideways.

Z Position (Vertical Movement):

The hexacopter ascends on both Earth and Titan, with Titan's ascent being less steep due to its lower gravity. This is consistent with a positive lift that exceeds gravitational pull.

Roll:

The roll angle increases over time in both simulations, which suggests that there is a persistent unbalanced moment causing the hexacopter to roll. This could be due to asymmetrical thrust, a misalignment in the rotors, or the absence of a roll stabilization mechanism.

Pitch:

The pitch angle shows a negative value increasing in magnitude over time for both Earth and Titan, which suggests the hexacopter is tilting forward more as time progresses. This is expected due to the simulation's control input that continuously increases pitch to simulate forward motion.

Yaw:

The yaw angle remains constant, indicating no rotation around the vertical axis, which is also expected due to the absence of yaw control inputs.

The simulations have revealed that while the hexacopter can generate lift and forward thrust, it exhibits roll instability that would need to be corrected in a real-world application. The pitch control appears to function as intended, tilting the hexacopter forward to promote forward motion. The continuous pitch down without stabilization could lead to an uncontrollable increase in forward speed and eventual descent. A control system implementation is imperative for practical flight stability and maneuverability.

CONCLUSION

We have explored the ideation, design, and simulation stages of an unmanned aerial vehicle (UAV) called TitanWing hexacopter, which is intended to be deployed on Titan, Saturn's most interesting moon. First on our adventure, we had to choose a suitable UAV configuration. After some research, we determined that a hexacopter was the best option because of its better lift capacity and redundancy in Titan's dense atmosphere.

Our research was primarily focused on understanding the behavior of the hexacopter in the different environments of Titan and Earth through rigorous simulation and dynamic modeling. We extrapolated the vehicle's translational and rotational dynamics through the lens of our models, revealing potential difficulties and performance subtleties in Titan's thick atmosphere. The simulations revealed a significant difference in the two celestial bodies' gravitational and atmospheric effects on the hexacopter's dynamics, highlighting the need for environment-specific design modifications.

According to our research, the hexacopter can provide enough lift and forward thrust, but in the absence of an active stabilization device, it rolls unstable. This is a crucial realization because it emphasizes how important it is to have sophisticated control systems to guarantee flight stability and maneuverability, which are essential for any practical use. Furthermore, even though the pitch control simulations were successful in advancing the hexacopter's forward motion, they also showed that the hexacopter tended to tilt forward more and more with time, requiring strong control techniques to reduce the possibility of an uncontrollable drop. All MatLab Simulations used can be found in the appendix.

To sum up, the TitanWing project has established a fundamental structure for upcoming investigations into UAV uses beyond Earth. In addition to offering insightful information about the aerodynamic viability of hexacopter flights in alien environments, our work has set the path for future technological developments. The implications of our research go beyond scientific discovery as we stand on the cusp of a new era of space adventure; they provide a glimpse into the future of interplanetary exploration and the potential role that unmanned aerial vehicles (UAVs) may play in helping us discover the mysteries of far-off worlds.

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